Marks : 40	SYJC March' 19 Subject : Maths – I Logic & Integration	Duration : 1.5 Hours. Set – B SOLUTION

Q.1. Attempt any One : (2 Marks each) (04)

1. Let p: $\sqrt{2}$
   Let q : 4 – 3i is a complex number
   Then the symbolic form of the given statement is $p \vee q$.
   The truth values of p and q are F and T respectively.
   $\therefore$ the truth value of $p \vee q$ is T. [$F \vee T \equiv T$]

2. Let p: 6 is an even number
   Let q : Pune is a harbour
   Then the symbolic form of the given statement is $p \vee q$.
   The truth values of p and q are T and F respectively.
   $\therefore$ the truth value of $p \vee q$ is T. [$T \vee F \equiv T$]

3. Let p : Re(z) < $|z|$, where z is a complex number.
   The truth value of p is T.
   Therefore, the truth value of $\sim p$ is F.

Q.2. Attempt any Four : (3 Marks each) (12)

1. Let p: Party's name must be given in a credit transaction
   Let q: single entry system is costly in a credit transaction.
   Then the symbolic form of the given statement is $p \wedge q$.
   The truth values of p and q are T and F respectively.
   $\therefore$ the truth value of $p \wedge q$ is F. [$T \wedge F \equiv F$]

2. (i) Yuvraj has sufficient money and he will buy a car.
   The symbolic form of given statement is $p \wedge q$)

p	q	$p \wedge q$
T	T	T

$\therefore$truth value of the given statement is 'T'.

(ii) If Yuvraj has sufficient money then he will not buy a car.
The symbolic form of given statement is $p \rightarrow \sim q$

p	q	$\sim q$	$p \rightarrow \sim q$
T	T	F	F

$\therefore$truth value of the given statement is 'F'.

(iii) Yuvraj does not have sufficient money or he will buy a car.
The symbolic form of given statement is $\sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T

$\therefore$truth value of the given statement is 'T'.

3. Let p: Every accountant is free to apply his own accounting rules.
Let q: Machinery is an asset
Then the symbolic form of the given statement is $p \leftrightarrow q$.
The truth values of p and q are F and T respectively.
$\therefore$ the truth value of $p \leftrightarrow q$ is F. $[F \leftrightarrow T \equiv F]$

4. The dual of the statement
$p \vee (q \vee r) \equiv p \wedge (q \wedge r)$

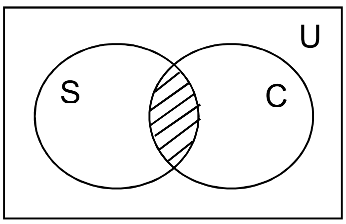
1	2	3	4	5	6	7
p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The entries in column 5 for $p \wedge (q \wedge r)$ and those in the column 7 for $(p \wedge q) \wedge r$ are identical. Hence, the statements $p \wedge (q \wedge r)$ and $(p \wedge q) \wedge r$ are logically equivalent.
$\therefore p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$.

5. Let p: Total assets minus capital is equal to liabilities
Let q: Book-keeping is the language of the business.
Then the symbolic form of the given statement is $p \vee q$.
Since, $\sim (p \vee q) \equiv \sim p \wedge \sim q$, the negation of the given statement is:
"Total assets minus capital is not equal to liabilities and book-keeping is not the language of the business."

6. L.H.S. $\equiv p \vee \{[\sim p \wedge (p \vee q)] \vee (q \wedge p)\}$
L.H.S. $\equiv p \vee \{[(\sim p \wedge p) \vee (\sim p \wedge q)] \vee (q \wedge p)\}$ (Distributive Law)
L.H.S. $\equiv p \vee \{[c \vee (\sim p \wedge q)] \vee (q \wedge p)\}$ (Complement Law)
L.H.S. $\equiv p \vee \{(\sim p \wedge q) \vee (q \wedge p)\}$ (Identify Law)
L.H.S. $\equiv p \vee \{(q \wedge \sim p) \vee (q \wedge p)\}$ (Commutative Law)
L.H.S. $\equiv p \vee \{q \wedge (\sim p \vee p)\}$ (Distributive Law)
L.H.S. $\equiv p \vee (q \wedge t)$ (Complement Law)
L.H.S. $\equiv p \vee q$ (Identify Law)
L.H.S. = R.H.S.

7. Let U : set of all human beings
Let S : set of all shareholders
Let C : set of all chartered accountant
Then the Venn diagram represents the truth of the given statement is given below :



$S \cap C \neq \phi$

Q.3. Attempt any One : (4 Marks each). **(04)**

1. $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 identical.

$\therefore (p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

2. $(\sim p \wedge \sim q) \wedge (q \wedge r)$

p	q	r	~p	~q	$\sim p \wedge \sim q$	$q \wedge r$	$(\sim p \wedge \sim q) \wedge (q \wedge r)$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	F
F	F	T	T	T	T	F	F
F	F	F	T	T	T	F	F

The entries in the last column of the above truth table are F.

$\therefore (\sim p \wedge \sim q) \wedge (q \wedge r)$ is a contradiction.

3. $(p \leftrightarrow r) \wedge (q \leftrightarrow p)$

p	q	r	$p \leftrightarrow r$	$q \leftrightarrow p$	$(p \leftrightarrow r) \wedge (q \leftrightarrow p)$
T	T	T	T	T	T
T	T	F	F	T	F
T	F	T	T	F	F
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	T	T	T

Q.4. Attempt any Two : (2 Marks each) **(04)**

1. $\int (3x + 4x)^3\, dx$

$= \dfrac{(3x+4)^4}{4 \times 3} + c$

$= \dfrac{(3x+4)^4}{12} + c$

2. $\int \dfrac{1}{1+\cos x}\, dx = \int \dfrac{1}{2\cos^2\left(\frac{x}{2}\right)}\, dx$

$= \dfrac{1}{2}\int \sec^2\left(\dfrac{x}{2}\right) dx = \dfrac{1}{2} x \dfrac{\tan(x/2)}{(1/2)} + c$

$= \tan \dfrac{x}{2} + c$

Alternative Method:

$\int \dfrac{1}{1+\cos x}\, dx = \int \dfrac{1}{1+\cos x} \, x \, \dfrac{1-\cos x}{1-\cos x}\, dx$

$= \int \dfrac{1-\cos x}{1-\cos^2 x}\, dx = \int \dfrac{1-\cos x}{\sin^2 x}\, dx$

$= \int \left(\dfrac{1}{\sin^2 x} - \dfrac{\cos x}{\sin^2 x}\right) dx$

$= \int \left(\operatorname{cosec}^2 x - \dfrac{1}{\sin x} \, x \, \dfrac{\cos x}{\sin x}\right) dx$

$= \int \operatorname{cosec}^2 x\, dx - \int \operatorname{cosec} x \times \cot x\, dx$

$= -\cot x - (-\operatorname{cosec} x) + c$

$= \operatorname{cosec} x - \cot x + c.$

3. Let $I = \int e^x \sec x (1 + \tan x) dx$

$= \int e^x (\sec x + \sec x \times \tan x) dx$

Put $f(x) = \sec x$

$\therefore \quad f'(x) = \sec x \cdot \tan x$

$\therefore \quad I = \int e^x [f(x) + f'(x)] dx$

$= e^x \times f(x) + c = e^x \times \sec x + c$

Q.5. Attempt any Four : (3 Marks each). **(12)**

1. $\int \dfrac{9x^4}{\sqrt{x}}\, dx + 5\int \dfrac{x^2}{\sqrt{x}}\, dx + \int \dfrac{1}{\sqrt{x}}\, dx$

$= 9\int x^{\frac{7}{2}}\, dx + 5\int x^{\frac{3}{2}} + \int x^{\frac{-1}{2}}\, dx$

$= 9\, \dfrac{x^{\frac{9}{2}}}{\frac{9}{2}} + 5\, \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2x^{\frac{9}{2}} + 2x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + c$

2. Let $I = \frac{(\sin x)}{(1 - \sin x)} dx$

$$I = \int \frac{\sin x(1+\sin x)}{(1 - \sin x)(1+\sin x)} dx$$

$$I = \int \frac{(\sin x + \sin^2 x)}{(1 - \sin^2 x)} dx$$

$$I = \int \left[\frac{\sin x + \sin^2 x}{\cos^2 x}\right] dx$$

$$I = \int \left[\frac{\sin x}{\cos x}\frac{1}{\cos x} + \frac{\sin^2 x}{\cos^2 x}\right] dx$$

$$I = \int \left[(\sec x)(\tan x) + \tan^2 x\right] dx$$

$$I = \int \left[\sec x \tan x + \sec^2 x - 1\right] dx$$

$$I = \int (\sec x \tan x) dx + \int (\sec^2 x) dx - \int (1) dx \quad I = \sec x + \tan x - x + c$$

3. $I = \int \frac{dx}{x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}}$

$$= \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\sqrt{3/2}} \tan^{-1} \left(\frac{x + 1/2}{\sqrt{3/2}}\right) + c$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}}\right) + c$$

4. $\int \frac{dx}{\sqrt{(x^2 + 4x + 4) + (5 - 4)}}$

$$= \int \frac{dx}{\sqrt{(x+2)^2 + (1)^2}}$$

$$= \log \left|(x+2) + \sqrt{(1)^2 + (x+2)^2}\right| + c$$

$$= \log \left|(x+2) + \sqrt{x^2 + 4x + 5}\right| + c$$

5. Let $I = \int_0^{\frac{\pi}{2}} \left(\cos^2 x\right) dx$

$$I = \int_0^{\frac{\pi}{2}} \left[\frac{1+\cos 2x}{2}\right] dx$$

$$I = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1)\,dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos 2x)\,dx$$

$$I = \frac{1}{2}[x]_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$I = \frac{1}{2}\left(\frac{\pi}{2} - 0\right) + \frac{1}{4}\left(\sin\pi - \sin 0\right)$$

$$I = \frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}(0 - 0)$$

$$I = \frac{\pi}{4}$$

6. $\int_1^2 \frac{dx}{x^2 + 6x + 5}$

$$= \int_1^2 \frac{dx}{\left(x^2 + 6x + 9\right) - 4}$$

$$= \int_1^2 \frac{1}{(x+3)^2 - (2)^2}\,dx$$

$$= \frac{1}{2(2)}\left[\log\left|\frac{x+3-2}{x+3+2}\right|\right]_1^2$$

$$= \frac{1}{4}\left[\log\left|\frac{x+1}{x+5}\right|\right]_1^2$$

$$= \frac{1}{4}\left[\log\frac{3}{7} - \log\frac{2}{6}\right]$$

$$= \frac{1}{4}\log\left(\frac{3}{7} \times \frac{6}{2}\right)$$

$$= \frac{1}{4}\log\left(\frac{9}{7}\right)$$

Q.6. Attempt any One : (4 Marks each). (04)

1. $\int 3\sqrt{x^2 - \frac{6x}{9} + \frac{4}{9}}\ dx$

$$= 3\int\sqrt{x^2 - \frac{6}{9}x + \frac{1}{9} + \frac{4}{9} - \frac{1}{9}}$$

$$= 3\int\sqrt{\left(x - \frac{1}{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= 3\left[\frac{\left(x - \frac{1}{3}\right)}{2}\sqrt{x^2 - \frac{6}{9}x + \frac{4}{9}} - \frac{1}{6}\log\left|\left(x - \frac{1}{3}\right) + \sqrt{x^2 - \frac{6x}{9} + \frac{4}{9}}\right| + c\right.$$

2. Let $I = \int \frac{x+4}{\left(x^2+6x+10\right)} dx$

$$I = \frac{1}{2}\int \frac{2x+8}{\left(x^2+6x+10\right)} dx$$

$$= \frac{1}{2}\int \left[\frac{(2x+6)+2}{\left(x^2+6x+10\right)}\right] dx$$

$$= \frac{1}{2}\int \frac{2x+6}{x^2+6x+10} dx + 1\int \frac{1}{x^2+6x+10} dx \quad \left[\frac{b}{a} = \frac{6}{1} \therefore \frac{b^2}{4a^2} = \frac{36}{4} = 9 \therefore \frac{b}{2a} = 3\right]$$

$$= \frac{1}{2}\int \frac{2x+6}{x^2+6x+10} + \int \frac{dx}{(x+3)^2+1^2}$$

$$= \frac{1}{2}\log\left|x^2+6x+10\right| + \frac{1}{1}\tan^{-1}(x+3) + c$$

$$= \frac{1}{2}\log\left|x^2+6x+10\right| + \tan^{-1}(x+3) + c$$

3. $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} dx$

$$= \int_0^1 \frac{x^2}{\sqrt{x}} dx + \int_0^1 \frac{3x}{\sqrt{x}} dx + 2\int_0^1 \frac{1}{\sqrt{x}} dx$$

$$= \int_0^1 x^{3/2} dx + 3\int_0^1 x^{1/2} dx + 2\int_0^1 x^{-1/2} dx$$

$$= \left[\frac{x^{5/2}}{5/2}\right]_0^1 + 3\left[\frac{x^{3/2}}{3/2}\right]_0^1 + 2\left[\frac{x^{1/2}}{1/2}\right]_0^1$$

$$= \frac{1}{5/2} + 3\left[\frac{1}{3/2}\right] + 2\left[\frac{1}{1/2}\right]$$

$$= \frac{2}{5} + 2 + 4$$

$$= \frac{2}{5} + 6$$

$$= \frac{32}{5}$$